```
> ### The 12 non-standard Jacobi primes to 10^14 of level at least 20.mws
```

Procedures

```
> with(numtheory): ### needed for 'order'
  ##########################################################################
  ######
  ### NEW procedures emerging at the weekend of Sat/Sun 9th/10th November 2013
  ### 01
  a_new := proc(p) local SOLN, s, a;
          SOLN := isolve(p = x^2 + 3*y^2):
             s := {op(op(1, [SOLN]))}:
             a := op(2, [op(op(1, s))]):
     if mods(a, 3) = -1 then a := a else a := -a fi:
             a; end:
  ### 02
  b_new := proc(p) local SOLN, s, b;
          SOLN := isolve(p = x^2 + 3*y^2):
             s := {op(op(1, [SOLN]))}:
             b := abs(op(2, [op(op(2, s))])):
     b; end:
  ### 03
  r_new := proc(p) local SOLN, s, a, b, r;
          SOLN := isolve(p = x^2 + 3*y^2):
             s := {op(op(1, [SOLN]))}:
             a := op(2, [op(op(1, s))]):
     if mods(a, 3) = -1 then a := a else a := -a fi:
             b := abs(op(2, [op(op(2, s))])):
     if b mod 3 = 0 then r := 2*a; elif mods(b, 3) = -1 then r := -(a + 3*b);
  else r := -(a - 3*b);
     fi; r; end:
```

```
### 04

u_new := proc(p) local SOLN, s, a, b, u;

        SOLN := isolve(p = x^2 + 3*y^2):

           s := {op(op(1, [SOLN]))}:

           a := op(2, [op(op(1, s))]):

   if mods(a, 3) = -1 then a := a else a := -a fi:

           b := abs(op(2, [op(op(2, s))])):

   if b mod 3 = 0 then u := 2*a; elif mods(b, 3) = -1 then u := -(a - 3*b);
else u := -(a + 3*b);

   fi; u; end:
>
```

The (twelve) non-standard Jacobi primes to 10^{14} at level at least 20

Here are all those Jacobis having minimum level 20 up to 10^{14}.

Such primes $\{p\}$ satisfy $p = 1$ (mod 3.2^{20}). Initially we computed to 10^{12}, then later we extended to 10^{14}.

There were:

1. 5 such primes to 10^{11}
2. 2 such primes between 10^{11} and 10^{12}
3. 3 such primes between 10^{12} and 10^{13}
4. 2 such primes between 10^{13} and 10^{14}

In fact the latter two Jacobis proved to be the only non-standard Jacobis between 10^{13} and 10^{14}

```
> print(``); bound := 20:

        L||bound := []:

for p from (3*2^bound + 1) by 3*2^bound to 10^12 do if isprime(p) then
L||bound := [op(L||bound), p]

fi od:
```

```
> nops(L20);
```

35871 **(2.1)**

```
> for p in L||20 do ordr||p := order(r_new(p), p):

      if factorset(ordr||p) = {2} then print(``); print(p, ifactor(ordr||p));
  fi

                  od:
```

$$69206017,\ (2)^{21}$$

$$270532609,\ (2)^{20}$$

$$1380974593,\ (2)^{20}$$

$$3221225473,\ (2)^{28}$$

$$3255828481,\ (2)^{20}$$

$$206158430209,\ (2)^{35}$$

$$844734922753,\ (2)^{21} \quad \textbf{(2.2)}$$

```
> p := 206158430209: ifactor(p-1);
```

$$(2)^{36}\,(3) \quad \textbf{(2.3)}$$

```
> p := 844734922753: ifactor(p-1);
```

$$(2)^{21}\,(3)\,(7)\,(19181) \quad \textbf{(2.4)}$$

- **Search from 10^{12} to 10^{14} at primes = 1 (mod 3.2^{20}):**

```
> 10^12                          mod 3*2^20;

  10^12 - 1380352                mod 3*2^20;

  10^12 - 1380352 + 3*2^20 + 1 mod 3*2^20;

  10^12 - 1380352 + 3*2^20 + 1;
```

1380352

0

1

1000001765377 **(2.5)**

```
> 1 000 001 765 377

> for p from 1000001765377 by 3*2^20 to 10^14 do if isprime(p) then

                   ordr||p := order(r_new(p), p):

      if factorset(ordr||p) = {2} then print(``); print(p, ifactor(ordr||p),
  ifactor(p-1)); fi

                                        fi od:
```

$$3788060491777,\ (2)^{25},\ (2)^{25}\,(3)\,(11)^2\,(311)$$

$$4754528796673,\ (2)^{30},\ (2)^{32}\,(3)^3\,(41)$$

$$6597069766657,\ (2)^{40},\ (2)^{41}\,(3)$$

$$25177098289153,\ (2)^{33},\ (2)^{33}\,(3)\,(977)$$

$$69803955978241,\ (2)^{31},\ (2)^{31}\,(3)\,(5)\,(11)\,(197)$$

(2.6)

```
> p; p mod 3*2^20;
```

100000000376833

1 **(2.7)**

The two new ones are the 14-digit (with levels at least 20, in fact levels 33 and 31):

```
> 25 177 098 289 153
```

```
> 69 803 955 978 241
```

```
> p := 25177098289153; a_new(p); b_new(p);
```

$p := 25177098289153$

5017481

25708 **(2.8)**

```
> p := 69803955978241; a_new(p); b_new(p);
```

$p := 69803955978241$

7505993

2118492 **(2.9)**