Fermat number record established in Ireland

A mathematically significant number of supra-astronomical size
was recently discovered in St. Patrick's College, Drumcondra.
One of its two discoverers - Dr. John Cosgrave - explains.

Pierre de Fermat (1601-1665), the outstanding French mathematician, has gained public recognition following the solution by Andrew Wiles of the famous problem of Fermat's 'Last Theorem.' Recounted in a widely acclaimed BBC Horizon programme, the related book by Simon Singh entered the best-sellers list.

Today there remains only one unresolved question of Fermat's, one of such *apparent* simplicity that on first hearing of it you might be tempted to exclaim: I can solve that!
However the consensus amongst mathematicians who have occupied themselves with this problem is that it may remain forever unresolved.

Fermat's question is, *what is the status of the 'Fermat numbers'?*, the Fermat numbers - $F_0, F_1, F_2, F_3, \ldots$ - being the unending sequence of numbers:

$$3, 5, 17, 257, 65537, 4294967297, 18446744073709551617, \ldots .$$

These numbers - formulated by Fermat in August 1640 - are formed in this simple way:

- starting from 2, a succession of numbers is formed by repeatedly squaring the previous one, producing: $2, 4, 16, 256, 65536, 4294967296, 18446744073709551616, \ldots$
- add 1 to each, forming: $3, 5, 17, 257, 65537, 4294967297, 18446744073709551617, \ldots$

The n^{th} Fermat number - F_n - is given by $F_n = 2^{2^n} + 1$, with $n = 0, 1, 2, 3, 4, 5, \ldots$. For example, $F_3 = 2^{2^3} + 1 = 2^8 + 1 = 257.$ Fermat believed *every F*-number to be prime (evenly divisible only by 1 and itself), but failed to prove it.

In 1732 the renowned Euler established that $F_5 = 2^{2^5} + 1 = 2^{32} + 1 = 4294967297 = 641 \times 6700417$, is evenly divisible by 641 and 6700417, establishing F_5 as the smallest *composite* (not prime) Fermat number. In the intervening years no other Fermat number has been identified as being prime, and there is now a general belief (not shared by the writer) that *every* Fermat number from F_5 onwards is composite.

F_5 to F_{23} are composite, but F_{24} (5,050,446 decimal digits), requiring a 47 by 47 feet surface to write it, allowing 4 digits per inch, is unresolved. A team led by Dr. Richard Crandall, has been attempting to establish its status (prime/composite?) for some time.

While F_{24} is large, it is insignificant compared to F_{382447}, found on July 24th in St. Patrick's College, Drumcondra, to be evenly divisible by $3 \times 2^{382449} + 1$ (115130 digits). This almost unimaginably large number - F_{382447} (over 10^{115136} digits) - would require a board measuring more than 10^{57550} by 10^{57550} *light years* to write it down, at 4 digits per inch, and even at 10^{30} digits per inch, the requisite board would measure at least 10^{57520} by 10^{57520} light years.

How has this result (a new world record for a Fermat composite, surpassing Jeffrey Young's F_{303088}, found at Silicon Graphics) been established? It has been made possible by two French contributions: a beautiful idea (1878) due to a self-taught farmer, François Proth (1852-79), together with the brilliant computer program (*Proth.exe*) of a contemporary scientist, Yves Gallot, who lives - by a lovely twist of fate - in Toulouse, where Fermat conceived the numbers now bearing his name.

A public lecture entitled 'The history of Fermat numbers from August 1640' - using **Maple** software, and demonstrating Gallot's program - will be given by the writer in St. Patrick's later this year; details will be available from his web site <www.spd.dcu.ie/johnbcos>. That site gives links to Gallot's program, to the dedicated compilation work of Dr. Ray Ballinger of Florida University and Dr. Wilfrid Keller of Hamburg University, to the writer's 3rd year undergraduate notes on Proth's 1878 theorem, and to much else besides.